MIND MAP : LEARNING MADE SIMPLE CHAPTER - 5

Complex Number & Quadratic Equations

Polar Representation

Let $a = r\cos\theta$

$b = r\sin\theta$

where,

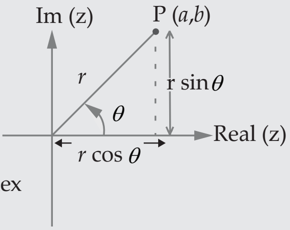
$r = |z|$

and θ = arg (z)

$\therefore z = a + ib = r(\cos\theta + i\sin\theta)$

The argument 'θ' of complex number z = a+ib is called principal argument of z if

$-\pi < \theta \leq \pi.$



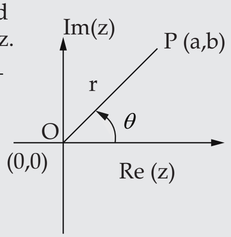
Modulus & Argument of Complex Number

If z= a+ib is a complex number

(i)Distance of z from origin is called as modulus of complex number z.

It is denoted by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Angle θ made by OP with +ve direction of X-axis is called argument of z.



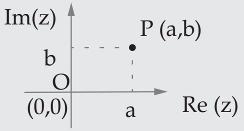
Powers of 'i'

$i = \sqrt{-1}, i^2 = -1$

In general, $i^{4k+r} = \begin{cases} 1; r = 0 \\ i; r = 1 \\ -1; r = 2 \\ -i; r = 3 \end{cases}$

Argand Plane

A complex number z=a+ib can be represented by a unique point P(a,b) in the argand plane



z=a+ib is represented by a point P (a,b)

Algebra of Complex Numbers

Let: z_1= a+*ib* and z_1= c+*id* be two complex numbers, where a,b,c,d ∈ R and $i = \sqrt{-1}$

1. Addition: $z_1 + z_2 = (a + ib) + (c + id) = (a + c) + i(b + d)$

2. Subtraction: $z_1 - z_2 = (a + ib) - (c + id) = (a - c) + (b - d)i$

3. Multiplication: $z_1 \cdot z_2 = (a + ib)\ (c + id)$

$= a(c + id) + ib(c + id)$

$= (ac - bd) + (ad + bc)i$

$(\because i^2 = -1)$

4. Division: $\frac{z_1}{z_2} = \frac{a + ib}{c + id} = \frac{a + ib}{c + id} \cdot \frac{c - id}{c - id}$

$= \left(\frac{ac + bd}{c^2 + d^2}\right) + \left(\frac{bc - ad}{c^2 + d^2}\right)i$

Note: If a+*ib* = c+*id*

⇔ a = c & b = d

Definition of Complex Numbers

A number of the form a+*ib*, where a,b ∈R and $i = \sqrt{-1}$ is called a complex number and denoted by 'z'.

$$z = \underbrace{a}_{\text{Real part}} + i\overset{}{b}\xrightarrow{}\text{Imaginary part}$$

Conjugate of a complex number: For a given complex number z=a+*ib*, its conjugate is defined as $\bar{z}$= a – *ib*

Solution of Quadratic Equation

- General form of quadratic equation in x is $ax^2+bx+c=0$,
  Where a,b, c ∈R & a≠0
  The solutions of given quadratic equation are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  $\because b^2 - 4ac < 0$

Note:
- A polynomial equation has atleast one root.
- A polynomial equation of degree n has n roots.

Multiplicative Inverse of complex Number

For a non-zero complex number z=a+ib $(a \neq 0, b \neq 0)$,

there exists a complex number $\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i$

denoted by $\frac{1}{z}$ or z^{-1}, called multiplicative inverse of Z

Such that: $(a + ib)\left(\frac{a}{a^2 + b^2} + i\frac{-b}{a^2 + b^2}\right) = 1 + 0i = 1$

Square root of Complex Number

Let $x + iy = \sqrt{a + ib}$, squaring both sides, we get $(x+iy)^2$= a+*ib* i.e. $x^2-y^2=a$, $2xy=b$ solving these equations, we get square root of z.